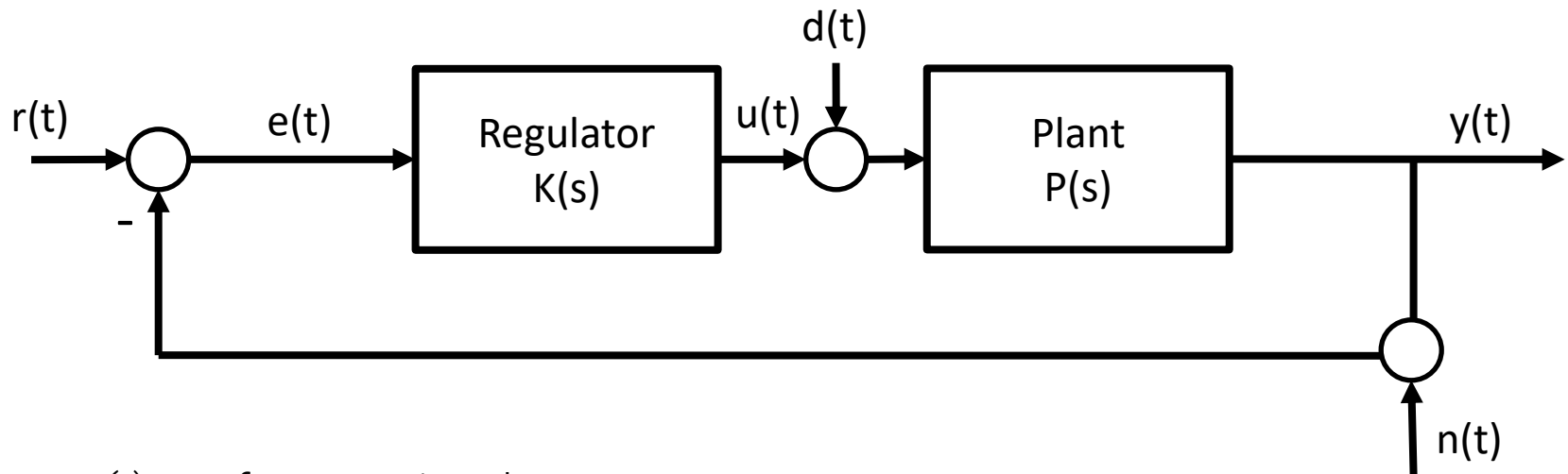


# Lecture #0.3

Standard industrial regulators

- Introduction to PID controllers
- PID control law:
  - Ideal
  - Real
- Some rules of thumb

# Reference scheme



- $r(t)$  – reference signal
- $e(t)$  – control error
- $u(t)$  – control action
- $y(t)$  – controlled output
- $n(t)$  – measurement noise
- $d(t)$  – additive (control action) disturbance

# Standard regulators

- The **dynamic characteristics** of controlled systems can vary greatly depending on the application
- However, it would be convenient to have controllers with a **standard structure**, to be configured according to the application
  - use controllers with a **fixed structure**
  - only tune some **parameters**

# Standard regulators

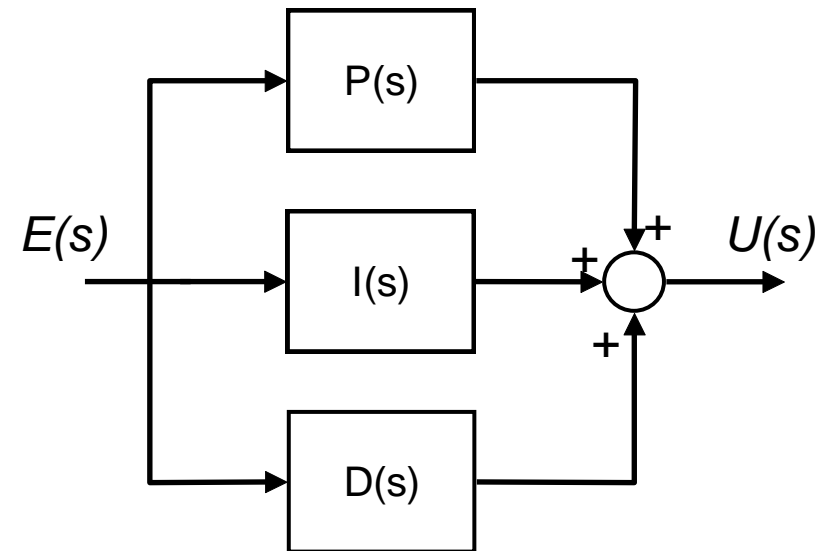
The most common **standard controllers** are

- LTI Controllers
  - Proportional Integral Derivative (PID)
- NL Controllers
  - Relay

# Industrial PID regulators

In a **PID regulator** the control action  $u(t)$  is obtained as the sum of three terms

1. The first is **proportional** to the error  $e(t)$
2. The second is proportional to the **integral** of  $e(t)$  (i.e. to its average value);
3. The third is proportional to the **derivative** of  $e(t)$



# Industrial PID regulators

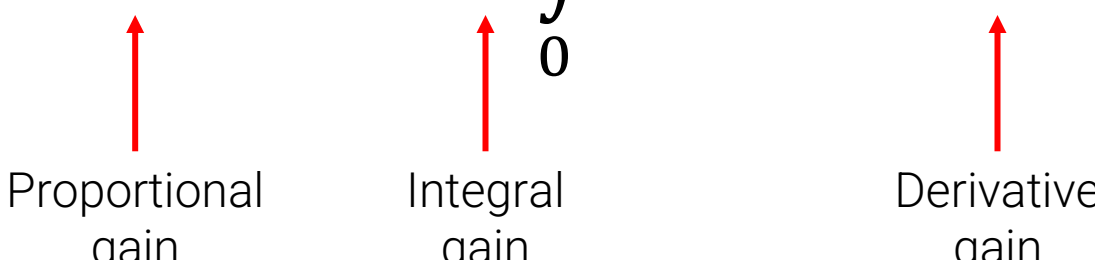
## Advantages

The success of PID regulators in the industrial field is essentially due to the following reasons:

- efficiently regulate **processes of different nature**  
(*thermal, mechanical, etc.*)
- realizable with **different types of technology**  
(*pneumatic, electronic, etc.*)
- convenience of a **standard structure**  
(*reduction of project costs, management and maintenance, benefits in the management of warehouse stocks*)
- often **do not require a detailed model** of the process  
(*auto-tuning procedures also exist*)

- In the time domain

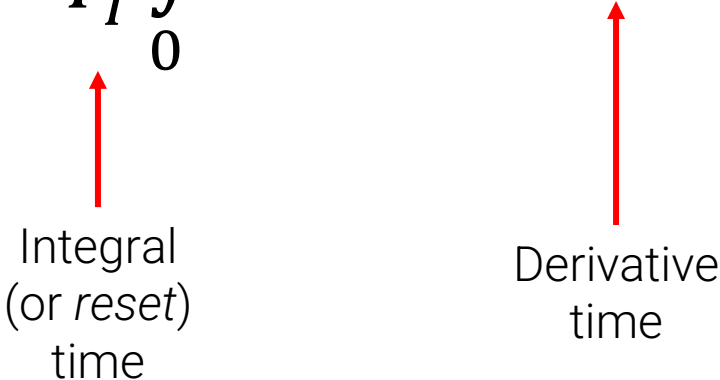
$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{de(t)}{dt}$$

  
Proportional gain      Integral gain      Derivative gain



- Another common representation:

$$u(t) = K_P \left( e(t) + \frac{1}{T_I} \int_0^t e(\tau) d\tau + T_D \frac{de(t)}{dt} \right)$$



Integral  
(or reset)  
time

Derivative  
time

# Ideal control law

In the Laplace domain

- **Ideal** control law: it is not physically achievable (**non-proper**)

$$U(s) = K_P \left( 1 + \frac{1}{T_I s} + T_D s \right) E(s) = K_P \left( \frac{T_D T_I s^2 + T_I s + 1}{T_I s} \right) E(s)$$

- **Real** control law obtained by **filtering the derivative action**

$$U(s) = K_P \left( 1 + \frac{1}{T_I s} + \frac{T_D s}{1 + s \frac{T_D}{N}} \right) E(s)$$

# Some rules of thumb

How does varying the parameters of a PID affect control performance?

# Proportional action

$$U_P(s) = K_P E(s)$$

- a high value of  $K_P$  reduces the **steady-state error** and can make the system's response **faster**
- On the other hand, a too high value of  $K_P$  may **destabilize** the system

# Proportional action

Steady-state error

- The residual steady state error can sometimes be handled with a **feedforward compensation** (e.g. resistive terms in electrical circuits)

$$\rightarrow U_P(s) = K_P E(s) + U_{FF}$$

# Proportional action

Effect of noise

- Moreover, computing the tf from the measurement noise  $n(t)$  to the output  $y(t)$

$$Y(s) = \frac{K_p P(s)}{1 + K_p P(s)} (R(s) - N(s))$$

- A high  $K_p$  increases the system's bandwidth and worsens the effect of measurement noise

# Integral action

$$U_I(s) = \frac{K_P}{T_I s} E(s)$$

- null steady-state error for constant  $r(t)$  and  $d(t)$
- 90° phase lag, bad for stability
- may cause actuator saturation (so-called integral *wind-up*)

# Derivative action

$$U_D(s) = \frac{K_P T_D s}{1 + \frac{T_D}{N} s} E(s)$$

- 90° phase lead, **good for stability**
- **amplifies the noise  $n(t)$  at high-frequency**, which may cause **actuator damage** for high control actions  $u(t)$   
→ choose N wisely!
- must be **cautious when dealing with discontinuous signals** (e.g. step reference)



# Overview

- In general not all control actions need be present
  - P
  - I
  - PI
  - PD
  - PID

	stability	$e_{\infty}$	$T_a$	overshoot
$K_P \uparrow$	Worsens	Decreases	Decreases	Increases
$T_I \downarrow$	Worsens	0 if $K_I \neq 0$	Decreases	Increases
$T_D \uparrow$	Improves	Not influenced	Not influenced	Decreases